Goethe University

Bayesian Modeling for Marketing Prof. Thomas Otter

Homework #3

Data augmentation, Probit, truncated normal prior, DAGs

1. Recall standard multivariate normal (regression-) theory, i.e.



define



then



(Either or both theta\_1 and theta\_2 can be vectors. When theta\_1 is a scalar, V\_11^-1 is simply the full conditional variance. If theta\_1 is a vector then it is the conditional variance-covariance matrix.)

1. Use this idea to create a version of “breg” in “rbprobitGibbsR.R” that draws individual elements of beta in a binary probit model conditional on all other elements.  
     
   breg1=  
   function(root,X,y,Abetabar)   
   {  
   #  
   # p.rossi 12/04  
   #  
   # Purpose: draw from posterior for linear regression, sigmasq=1.0  
   #   
   # Arguments:  
   # root is chol((X'X+A)^-1)  
   # Abetabar = A\*betabar  
   #  
   # Output: draw from posterior  
   #   
   # Model: y = Xbeta + e e ~ N(0,I)  
   #  
   # Prior: beta ~ N(betabar,A^-1)  
   #cov=crossprod(root,root)  
   betatilde=cov%\*%(crossprod(X,y)+Abetabar)  
   **betatilde+t(root)%\*%rnorm(length(betatilde))**  
   }  
     
   Specifically, you need to replace this function’s last line (in bold) with a loop over beta elements, drawing one beta-element at a time conditioned on all other beta-elements, respectively. Note that you will need the old beta state as an input to this function because Gibbs-cycling through individual beta elements beta requires starting values.   
     
   Add this function to “rbprobitGibbsR.R” and demonstrate that drawing from the full conditional distributions of individual beta elements is numerically less efficient than drawing from the (conditional) joint distribution of all beta-elements, i.e., than drawing the vector of betas directly, conditional on augmented latent variables {z} as in “breg”. (You may want to add an argument to your updated “rbprobitGibbsR.R” that defines which version of “breg” should be used in a particular run.)
2. Next, take advantage of your ability to fully conditionally update elements of beta from univariate distributions to impose sign constraints using a truncated normal prior. That is, you will create yet another version of “breg” that features the full conditional updates you created under a), and on top allows for convex constraints on elements of the beta vector.

Test and compare the different versions of your code, i.e., summaries of posterior draws from the different versions using the simulation settings given in Hw3.R

1. Draw the directed acyclic graph (DAG) corresponding to the following data-generating mechanism:



1. From the DAG, derive the full conditional distributions required for augmenting the z’s in abstract notation. Then think about the respective “other z’s” role in the update of z\_1, z\_2, z\_3. What information is transmitted from the respective other z’s in these updates?
2. Derive the conditional distribution required for updating the beta-vector in abstract notation. In what (regression-)form is the (conditional) likelihood for the betas?